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Loss Aversion Distribution: The Science Behind Loss Aversion Exhibited by Sellers of Perishable Good

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Abstract This research introduces the concept of the loss aversion distribution, a pioneering framework designed for the analysis of consumer behavior. Departing from the conventions of traditional exponential models, this innovative approach incorporates a non-memoryless characteristic, which modulates the consumer's response to loss aversion throughout the product's life cycle. This modulation is achieved by a variable exponent influenced by the parameter b, representing the psychological impact of loss aversion, and the constant k, which reflects the market value of the good at the time of manufacture. Together, these parameters adeptly encapsulate the dynamic nature of consumer loss aversion from the moment of manufacture to the point of expiry. The model elucidates an initial muted response from consumers at the onset of ownership, which then intensifies during the mid-life cycle of the product, before ultimately diminishing as the product approaches its expiry. Through a meticulous derivative analysis of the probability density function, the study delineates the distribution's key properties, including its monotonicity, boundedness within the interval [0,1], and its adherence to non-negativity. This framework not only enhances our comprehension of consumer behavior in relation to perishable goods but also paves the way for further investigations into psychometrics and the intricacies of loss aversion modeling.

Keywords: Loss aversion · Prospect theory · Probability theory

1 Introduction

The study of loss aversion has been around for decades since the 1970s. Psychologists Daniel Kahneman and Amos Tversky first introduced the concept of loss aversion in their seminal paper titled, "Prospect Theory: An Analysis of Decision under Risk" (Kahneman & Tversky, 1979), and since then many researchers have expanded this theory beyond decision science and cognitive psychology. Ariely, Huber, and Wertenbroch (2005) explored two key constructs, emotional attachment, and cognitive perspective, which defined 'the boundaries of loss aversion', while Morewedge and Giblin (2015) provided the explanation of Attribute Sampling Bias, which is a collective explanation of the valuation gap in addition to the loss aversion. Researchers have also shifted their focus from product-based research to include the change in the role of consumers to become sellers from the perspective of economic concepts such as the willingness-to-sell and willingness-to-pay constructs (Simonson & Drolet, 2004). In recent years, Gal and Rucker (2018) argued that losses loom larger than gain when consumers choose riskier options in investment to avoid potential loss in missing out on the opportunity to earn more. When consumers sell their possessions which can come in the form of goods or services, they exhibit a loss in ownership which is 'a manifestation of loss aversion.' Kahneman and Tversky (2013b); Tversky and Kahneman (1991). These concepts and theories support the basis of this research paper.

However, at this moment, we do not have a quantitative method to understand loss aversion, particularly for scenarios whereby products that are time-sensitive and value-depreciating are involved in the trade. These time-sensitive and value-depreciating products are commonly known as 'perishable products'. Perishable products refer to goods that are subject to quality degradation, loss of value, or spoilage over a period due to biological decay, environmental conditions, or the passage of time. This category encompasses a broad range of items, including but not limited to, foodstuffs (such as fruits, vegetables, meat, and dairy products), pharmaceuticals, certain chemicals, and biological samples. The perishability of these products imposes critical constraints on their handling, storage, and distribution, necessitating optimized supply chain strategies to minimize waste, ensure quality preservation, and maximize economic viability. The probability distribution potentially explaining loss aversion is the exponential distribution, within the gamma distribution family. However, when consumers initially acquire a product, the decay in value does not follow the uniform pattern observed in the exponential distribution. Instead, the initial phase of loss aversion, should they decide to sell, typically mirrors a lossless behavior, succeeded by a gradual exponential increase in loss aversion, and then a gradual exponential decrease, culminating in a plateau as the expiry date approaches. For instance, when considering the resale of a Swiss chocolate bar manufactured just a day prior, we do not calculate its selling price by linearly extrapolating the days until expiry to determine its value. Rather, we tend to adhere to the purchase price during the initial days of ownership. Consequently, instead of selling at \$28 when the market price is \$30, we are more inclined to sell at \$30 on the day following the first day of ownership. This behavior is prevalent in the early stages of ownership; for example, we are more likely to sell at \$30 rather than at a significantly lower value when the expiry date is one year away and the product was manufactured a month ago. The exponential distribution fails to capture this specific behavior. Furthermore, the concept of hyperbolic discounting, as supported by economists and psychologists, does not address the nuances of consumer loss aversion in the early stages of ownership. Therefore, we propose a new form of the generalized exponential distribution, which we term the "loss aversion distribution.

2 Literature Review of Loss Aversion

Loss aversion is characterized as an asymmetric valuation phenomenon, where the displeasure from relinquishing an object is stronger than the pleasure derived from acquiring it. This concept, first detailed by Kahneman, Knetsch, and Thaler (1991), emerges from the disparity between what consumers are willing to relinquish and the price they are prepared to pay to retain a good, as explained by Knetsch and Sinden (1984). It is identified as a widespread occurrence (Novemsky & Kahneman, 2005) and stems from a heightened sensitivity to perceived losses compared to equivalent gains, predominantly motivated by the more urgent need to address pain over pleasure (Kahneman et al., 1991; Tversky & Kahneman, 1991). The way options are presented, either as losses or gains, significantly affects decision-making, with this framing effect becoming more pronounced with age during adolescence (Williamson, MacDonald, & Brosnan, 2019).

The principle of loss aversion is noticeable when spending on essentials is proportionally higher than on non-essentials (Wicker, Hamman, Hagen, Reed, & Wiehe, 1995), highlighting the influence of reference points in decision-making processes (Mellers, Yin, & Berman, 2021; Schurr & Ritov, 2014). Factors such as the similarity of exchanged items (Hanemann, 1991), goods offering identical benefits (Novemsky & Kahneman, 2005), rejection costs (Kahneman & Tversky, 2013a; Thaler, 1999), and items of minor value that do not significantly affect a consumer's wealth or circumstances (Gal & Rucker, 2018) also impact loss aversion. However, it does not apply to straightforward exchanges, like swapping a $20 bill for another, items of minimal value, decisions by experts, or choices involving high moral responsibility (Aggarwal, Zhang, Iacobucci, & Nowlis, 2006; Bateman, Kahneman, Munro, Starmer, & Sugden, 2005; Bettman & Sujan, 1987; Boyce, Brown, McClelland, Peterson, & Schulze, 1992; Brenner, Rottenstreich, Sood, & Bilgin, 2007; Harinck, Dijk, Van Beest, & Mersmann, 2008; Novemsky & Kahneman, 2005).

The psychological aspect of consumers also plays a critical role in loss aversion. For instance, consumers experiencing a strong self-connection to a lost possession face more significant distress and negative emotions (Ferraro, Escalas, & Bettman, 2011). Preferences not to compute gains and losses (Aggarwal et al., 2006) and decisions aimed at minimizing anticipated negative feelings (Anderson, 2003) reflect loss aversion's psychological underpinnings. From a social psychology viewpoint, individuals strive for a balance between gains and losses (Clark & Mills, 1993) and may opt for riskier investments to avoid the loss of potential earnings (Gal & Rucker, 2018), especially when considering options collectively rather than individually (Thaler, 1999).

Loss aversion manifests in two forms: valence loss/gain and possession loss/-gain (Brenner et al., 2007; Thaler, 1999), with consumers evaluating gains and losses based on remembered ordinal measures (Walasek & Stewart, 2014). Decision-making challenges arise from the difficulty in retrieving information from memory and handling complex tasks under stress (Payne, Bettman, & Luce, 1996), further complicated by hindsight bias, known as the "knew-it-all-along" effect

(Hawkins & Hastie, 1990). While loss aversion has been assumed universal in research, its application necessitates contextual understanding (Gal & Rucker, 2018). Debates around its existence and the exploration of its limits are encouraged, with suggestions to retain loss aversion as a concept alongside the psychological law of inertia to explain valuation gaps more comprehensively (Gal & Rucker, 2018; Johnson & Meyer, 1984). The framing of valuation gaps significantly influences choice valuation (Tversky & Kahneman, 1991), with consumers weighting differences more heavily when perceived as disadvantageous. This phenomenon affects both risky and riskless choices, with the emotional attachment to goods and money playing a role in loss aversion's intensity (Gächter, Johnson, & Herrmann, 2022).

Loss aversion also impacts the evaluation of goods' multiple attributes, such as pricing and quality, leading to complex decision-making processes and preferences based on prominent attributes over matching different ones, known as the prominence hypothesis (Tversky, Sattath, & Slovic, 1988). The cognitive ease of attribute-based processing underscores its preference in decision-making (Russo & Dosher, 1983), while the complexity of trade-offs and choice deferral in the face of conflict highlight the nuanced implications of loss aversion in consumer behaviour (Frisch & Clemen, 1994; Huber & McCann, 1982; Tversky & Shafir, 1992).

In this study, we advance the existing body of research by merging the concept of loss aversion which is pivotal within the realm of behavioral economics with probability theory—a fundamental aspect of data science. Our objective is to employ an innovative methodology for the analysis of consumer behavior, with a specific focus on the inherent irrationality of consumer actions. To this end, we tap on probability theory, particularly the probability density function, for a quantitative assessment of these actions. While previous studies have primarily concentrated on the theoretical underpinnings of loss aversion as discussed in our literature review, our research seeks to operationalize the willingness-to-accept (WTA) framework derived from the loss aversion theory. We intend to apply this framework to practical scenarios in the business sector, thereby bridging the gap between theoretical concepts and real-world applications.

3 Literature Review of Probability Distributions

Probability distributions are characterized by their density functions, such that the area under the curve for a probability density function (PDF) is equal to 1, ensuring that the total probability is distributed across the range of possible outcomes (Ross, 2014). They reflect the blend of deterministic events, with added stochastic phenomena which is attributed to chance (Papoulis & Unnikrishna Pillai, 2002). Hence, it is a distribution of chance and non-chance or pattern, and the $f(x)$ is the output at which the chance or non-chance occurred. This output is expressed in a probability figure.

The exponential distribution is characterized by its simplicity and memoryless property. The probability of an event in the future does not depend on how much time has already elapsed (Klugman, Panjer, & Willmot, 2012), and it is mainly suitable for modeling time between independent events that happen at a constant rate, a scenario commonly encountered in the fields of reliability engineering and queuing theory (William & Escobar, 1998). Given the assumption of a constant rate, the PDF of the exponential distribution is often characterized by a single parameter that impact the way how the probability of events is derived over the numerical space (Klugman et al., 2012). This parameter not only dictates the average rate at which events occur but also shapes the exponential curve, thereby providing a versatile tool for statistical analysis in various scientific and engineering domains, assessing risks and uncertainties in complex systems (Rubinstein & Kroese, 2016).

The probability density function (PDF) of the exponential distribution is given by:

$$f(x;\lambda) = \lambda e^{-\lambda x};\ x \geq 0; \lambda > 0.$$

The cumulative density function (CDF) of the exponential distribution is given by:

$$F(x;\lambda) = 1 - e^{-\lambda x};\ x \geq 0; \lambda > 0.$$

In recent years, researchers have introduced the generalized exponential distribution to provide more flexibility in data modeling with non-constant rates or more complex patterns (Gupta & Kundu, 1999). One notable work is Gupta and Kundu (2001). The Generalized Exponential Distribution (GED) is characterized by its two parameters, one of which α shapes the distribution and the λ scales the distribution. The probability density function (PDF) is given by:

$$f(x;\lambda) = \alpha\lambda \left(1 - e^{-\lambda x}\right)^{\alpha-1} e^{-\lambda x};\ \alpha, \lambda, x > 0,$$

and the cumulative density function (CDF) is given by:

$$F(x;\lambda) = \left(1 - e^{-\lambda x}\right)^{\alpha};\ \alpha, \lambda, x > 0.$$

Our research differs from the flexible Generalized Exponential Distribution. First, the GED employs a dual-parameter approach (α and λ), making it versatile in fitting data but also more complex in terms of parameter estimation and interpretation. Our research equation, however, opts for a single-parameter model focused on b, aiming for simplicity and interpretability while still attempting to capture complex behavior patterns. This difference in parameterization directly influences the ease of optimization and the clarity of the model's predictions. Second, the GED's functional form is designed to capture a wide range of distribution shapes through its parameters, enabling flexibility in modeling different types of data distributions. In contrast, our equation incorporates an innovative exponentiation mechanism where the effect of b is modulated in a unique manner by interacting with $f(x)$ and inversely with $\sqrt{x}$. This distinct approach allows our model to explore specific aspects of economic behavior, such as the impact of time until expiry and loss aversion, with a targeted focus that the GED's general form might not specifically address. Last, our research equation explicitly considers the temporal aspect (through the modification by x and its interaction with b and k) in a way that the GED does not. This consideration allows for a nuanced examination of how the proximity to goods' expiration influences economic decisions, a factor of particular relevance in studies of loss aversion and decision-making processes.

Beyond merely integrating loss aversion with probability theory, our research endeavors to enrich probability theory itself by elaborating on the concepts found within the generalized exponential distribution family. By doing so, we aim to inspire further research into the inherent traits of human behavior, advocating for their quantification through statistical methodologies within the data science field. This dual approach not only contributes to a deeper understanding of behavioral economics but also advances the mathematical tools available for analyzing complex human behaviors.

4 Loss Aversion Sensitivity (LAS) Function

The LAS function was first introduced at a research conference in Zagreb by Koh (2022). The motivation was to create a new class of functions that could potentially explain loss aversion exhibited by consumers when it comes to selling the perishable goods. The function, shown in Equation (1) is similar to the exponential function, except for the initial part of the function whereby the starting point is at $f(0)$ at the origin, followed by a very gradual acceleration in a downward curve, after which it accelerates quickly to the mid-point which is the halfway point between manufactured date and expiry date, and then plateaus off when it is near $f(x) \approx 0$.

$$\phi(x;b) = \left(k * e^{-ek^{-b/\sqrt{x}} * x}\right) \ \forall\ k, x, b \in \mathbb{R}; k, x, b > 0. \tag{1}$$

Just as the exponential function demands, the function value approaches infinity as the events expand.

In the dissertation published by Koh (2023), a sample of $n = 385$ is taken from the Singaporean millennial population of aged 25 to 36 years old. The respondents were either Singaporean citizen or permanent resident, male or female, residing in Singapore, and their ethnic race can be Chinese, Malay, Indian, Eurasian, and others. The author deployed a quantitative survey via an online poll service, with a sampling size taken from a 1.2 million millennial population in Singapore, confidence level at 95% with a margin of error of 5% or less.

In the author's research, the respondents were asked for their selling prices of a bar of chocolate at different points in the time of possession, from the start of the ownership to the day it expires. A LOESS method was applied to the collected data to show the robustness of the NLS curve fitting. 74% of all the respondents followed the proposed theoretical model introduced by Koh (2022), with a standard error of median value at 0.068 and mean value at 0.0887, ranging from 0.0176 to 0.948 (< 1). We have a strong basis to believe that the LAS function can be transformed into a probability distribution, providing one step closer to explaining loss aversion using science. The proposed b-parameter ranged from 1.05 to 7.12 for the chocolate bar priced at around \$20.

In this research, we propose a significant modification to enhance its descriptive power concerning the seller's behavior in the face of loss aversion. First, we address the modifier $-ek^{-b/\sqrt{x}} * x$, focusing on the removal of Euler's number (e) from the modifier. Our hypothesis at this moment posits that the presence of Euler's number within this context does not add substantial explanatory value to the phenomenon of loss aversion among sellers. Second, we reconsider the constraints on the parameter b, proposing that b should now be considered without the previously imposed constraints. This adjustment aims to explore a broader range of behaviors and responses encapsulated by the model. We therefore propose a change in the formulation of the LAS function as such:

$$\phi(x; b) = \left(k * e^{-k^{-b/\sqrt{x}} * x}\right) \ \forall\ k, x \in \mathbb{R}; k, x, b > 0.$$

5 Loss Aversion Distribution

The Loss Aversion Distribution is a new type of generalized exponential distribution. Due to its non-memorylessness of the function, it does not fall under the exponential distribution family. It takes into account the parsimonious behavior exhibited by consumers for selling perishable goods and a bivariate correlation between the market price of the goods at the time of manufacturing and the time of expiry. However, unlike the exponential distribution, it does not assume memorylessness, an event that occurs influences the next, such that a good that is expected to expire in 1 year time will have a day-to-expire of 6 months after 6 months have passed at a given constant rate. Given this non-memorylessness property of this distribution, it becomes very versatile for modeling data that are predicted to happen. The challenge becomes the prediction of the function at each point in time, given that human behavior is predictably consistent but irrational. It is this irrationality shown by a non-constant rate in loss aversion coupled with the predictive consistency that gives rise to this new type of distribution.

Following the LAS function introduced before, the Loss Aversion Distribution has the following probability density function and cumulative density function. First, we find the starting point of the curve k where $k \in \mathbb{R}; k > 0$. The constant k signifies the value of a product at the time of manufacturing. Following this, we create an exponent to reflect the exponential rate in decreasing trend, such that the curve trends downward toward the axis $k * e$. Next, we characterize the raised exponent by the constant k and the variable x:

$$k * e^{-kx}. \tag{2}$$

In Equation 2, we examine the transformation of market price at the point of manufacture, modulated by the constant e, representing the Euler's number, and the variable x. However, it is imperative to establish that k initiates at $x = 0$, implying that negative values of x are not considered, consequently,

$$k * e^{-k^{-\frac{1}{\sqrt{x}}} * x}.$$

At this juncture, the function's value initiates at $f(x) = k$ for $x = 0$. Incorporating the square root of x guarantees that the function remains non-negative across its domain. Following this, a scaling parameter is introduced to adjust the magnitude of the curve:

$$C = \int_0^\infty k * e^{-k^{-b/\sqrt{x}} * x} \; ; k, x \in \mathbb{R} \; ; k, b, x \geq 0. \tag{3}$$

The parameter b functions as a scaling factor for the exponent, establishing a direct relationship with the constant k to facilitate scaling. This implies that the magnitude of the exponential term e is modulated by the value of k. Such a relationship is particularly relevant in contexts where loss aversion exhibits a proportional correlation with the value of the perishable product. Moreover, the incorporation of the square root of x serves to adjust the parameter b proportionally, thereby ensuring that loss aversion is appropriately modulated with respect to time until expiry. Equation 3 serves as the Normalization Constant C for the probability density function.

It is worth noting that when $b = 0$, the function becomes the exponential function:

$$C = \int_0^\infty k * e^{-x} \; ; k, x \in \mathbb{R} \; ; k, x \geq 0.$$

Note that this is an exponential function and not the exponential distribution which assumes memorylessness.

6 Probability Density Function and Cumulative Density Function

A probability density function (PDF) represents the relative likelihood for a continuous random variable to take on a given value. It is a fundamental concept within the probability theory. The proof for the integral of $f(x)$ equal 1 is provided by the utilization of the Normalization Constant C. The verification that the values of the probability density function (PDF) reside within the [0,1] interval is achieved through the application of the Normalization Constant as seen in Equation 4. This constant ensures that the integral of the PDF over its entire domain equals 1, thereby satisfying the fundamental requirement for all probability values across the defined domain to sum to unity. The probability density function for the loss aversion distribution is given by:

$$f(x) = \frac{k * e^{-k^{-b/\sqrt{x}} * x}}{C} \; ; k, x \in \mathbb{R}; k, x \geq 0.$$

The cumulative density function for loss aversion distribution is given by:

$$F(x) = \int_0^{\infty} \frac{k * e^{-k^{-\ b/\sqrt{x}}\ *\ x}}{C}\ ; k, x \in \mathbb{R}; k, x \geq 0. \quad (4)$$

The loss aversion distribution must adhere to the rule of non-negativity and monotonicity, along with the assumption that the probability density function does not exhibit memorylessness. Non-negativity ensures that the values of the probability density function (PDF) and cumulative distribution function (CDF) are always greater than or equal to zero, reflecting the principle that probabilities cannot be negative. Monotonicity, particularly for the CDF, means that the function must be non-decreasing as the value of x increases, ensuring that probabilities accumulate in a manner consistent with the intuitive understanding of probability. This characteristic ensures that the CDF represents a true accumulation of probabilities up to a certain point, which aligns with the foundational principles of probability theory.

Moreover, the lack of memorylessness in the loss aversion distribution differentiates it from certain distributions like the exponential distribution, where the future probability distribution is independent of the past. This assumption is crucial for modeling loss aversion, as the psychological impact of losses and gains is not uniform and can depend on previous outcomes or the context of the decision-making environment. In distributions exhibiting memorylessness, the probability of an event occurring within a certain time frame remains constant, regardless of how much time has already passed. However, in the context of loss aversion, past experiences and the magnitude of potential outcomes can significantly influence decision-making processes, requiring a more complex model that accounts for these factors.

6.1 Non-Negativity

The exponential function, denoted as $e^{-k^{-\ b/\sqrt{x}}\ *\ x}$ retains positivity for all $k, x \geq 0$ regardless of the specific values of k and x. This is because the base of the exponential function, e, is a positive constant, and the exponent of it is defined in such a way that results in a negative value, ensuring that the output is always non-negative. $k^{-\ b/\sqrt{x}\ *\ x}$ is also positive since the condition restrains k and x to be positive and non-zero, because k raised to any real power remains positive and multiplying by the variable does not change the positivity. The inverse exponent guarantees that the exponential function remains non-negative. Consequentially, we arrive at an equation whereby positive divided by positive of $f(x) \geq 0$ for all $k, x \geq 0$. Therefore, we have proven that the probability density function is non-negative and for any given x, the distribution yields a $f(x)$ non-negative probability intervals over values of non-negative x.

6.2 Monotonicity

The derivative of the probability density function demonstrates that the parameter b is modulated by both x and k, indicating a dependence where b's influence on the function's rate of change is dynamically adjusted by the variable x. Taking the first derivative of the density function, we have

$$\begin{aligned} f^{'}(x) &= \frac{d}{dx}\left(\frac{k * e^{-k^{-\frac{b}{\sqrt{x}}} * x}}{C}\right) \\ &= \frac{k}{C} * \frac{d}{dx}\left(-k^{-\frac{b}{\sqrt{x}}} * x\right) \\ &= \frac{k}{C} * \left[\left(-k^{-\frac{b}{\sqrt{x}}} * \ln(k) * \frac{bx}{2x^{\frac{3}{2}}}\right) - \left(k^{-\frac{b}{\sqrt{x}}}\right)\right] \\ &= \frac{k}{C} * \left[\left(-k^{-\frac{b}{\sqrt{x}}} * \frac{\ln(k) * b}{2\sqrt{x}}\right) - k^{-\frac{b}{\sqrt{x}}}\right] \end{aligned} . \quad (5)$$

In the simplified expression of the derivative in Equation 5, there are two primary components to consider. The first term, $-k^{-b/\sqrt{x}} * \ln(k) * b/(2\sqrt{x})$ combines an exponential decay factor influenced by k and b, and modulated by x through $-b/\sqrt{x}$ in the exponent and the $1/\sqrt{x}$ in the coefficient. This term represents how the rate of change of the function decreases more slowly as x increases, due to the presence of x under a square root in the denominator, indicating that the impact of x on the function's rate of change diminishes as x grows larger. The second term, $-k^{-b/\sqrt{x}}$, acts as a foundational exponential decay factor, similarly influenced by x through the exponent $-b/\sqrt{x}$. This term signifies the overall trend of the function as it decays, with the rate of decay modulated by the values of k and b. The presence of x in the exponent's denominator means that as x increases, the rate of decay becomes less steep, illustrating a decrease in the function's rate of change.

Since both terms $-k^{-b/\sqrt{x}} * \ln(k) * b/(2\sqrt{x})$ and $k^{-b/\sqrt{x}}$ are subtracted, the derivative function is negative, indicating the function it derives from is monotonically decreasing across its domain so long as $k, b, x > 0$.

6.3 Moments of Probability Density Function

The first moment of a PDF which is the mean value is calculated as the integral of the product of x multiplied by the PDF overs its domain. Unlike the exponential distribution, the rate parameter is replaced by $k^{-b/\sqrt{x}}$. Given that the rate parameter is dependent on x, it's not constant across the distribution. Hence, we express the dth moment as:

$$S = \int_0^\infty x^d \left(\frac{k * e^{-k^{-\frac{b}{\sqrt{x}}} * x}}{C}\right) dx.$$

6.4 Non-Memorylessness Property of the Distribution

Memorylessness is an evaluation of the function that assumes the future interval δx depends only on δx and not on x. The equation is given by:

$$\begin{aligned}\frac{f(x+\delta x)}{f(x)} &= \frac{k * e^{-k^{-\frac{b}{\sqrt{x+\delta x}}} * (x+\delta x)}}{k * e^{-k^{-\frac{b}{\sqrt{x}}} * x}} \\ &= e^{\left(-k^{-\frac{b}{\sqrt{x}}} * x\right) - \left(-k^{-\frac{b}{\sqrt{x+\delta x}}} * (x+\delta x)\right)} . \\ &= e^{\left(-k^{-\frac{b}{\sqrt{x}}} * x\right) + k^{-\frac{b}{\sqrt{x+\delta x}}} * (x+\delta x)}\end{aligned} \tag{6}$$

For a function to be memoryless, the simplification of the exponent in Equation 6 must depend on δx and not on x. However, in the given equation, the expression inside the exponent clearly depends on both the δx and x and it also involves $x + \delta x$ within a complex interaction of exponential and power functions, modulated by b parameter; k is a constant. The presence of x in the final expression demonstrates that the ratio $f(x + \delta x)/f(x)$ cannot be simplified to a function that depends only on δx. Hence, our assumption that $f(x)$ could be memoryless leads to a contradiction and we conclude that the function does not possess the memoryless property.

6.5 Calibrating b-parameter

In this section, we delve into the methodology developed for the precise calibration of the b parameter within a probabilistic framework. This parameter significantly influences the temporal dynamics of the model's cumulative distribution function (CDF). The objective is to align the CDF at a specific temporal marker, $x = days\_to\_expiry$, with a target probability, optimizing the model for predictive accuracy and alignment with empirical data.

We provide an R function, shown below, to calibrate b. This method optimizes a parameter, b, by minimizing the error between the model's CDF and a target value, set close to unity minus the smallest positive number representable in the computing environment. Utilizing an iterative approach, the algorithm adjusts b based on an initial guess, employing an error function that computes the absolute difference from the target CDF. The process involves an adaptive step sizing, where the step size is halved and the direction is reversed upon increasing error, iterating until the error change falls below a specified tolerance. This approach, while straightforward, offers a practical solution for rapid parameter estimation in scenarios where bounded intervals may not be feasible.

```
find_b_for_CDF_target <- function(k0, initial_guess =
   50, max_iterations = 1000, tol = 1e-6) {

  # Target CDF value set to 1 minus the smallest
     positive number
```

```
  target_CDF <- 1 - .Machine$double.xmin

  # Error function to compute the absolute difference
     from the target CDF
  error_function <- function(b) {
    abs(F(days_to_expire, b, k0) - target_CDF)
  }

  # Initial guess for the optimization
  b_current <- initial_guess
  error_current <- error_function(b_current)

  # Initial step size, can be adjusted based on
     specific problem characteristics
  step_size <- 1

  for (i in 1:max_iterations) {
    # Attempt to adjust b in the positive direction
    b_next = b_current + step_size
    error_next = error_function(b_next)

    # If error increases, reverse and halve the step
       size
    if (error_next > error_current) {
      step_size = -step_size / 2
      next
    }

    # Update the current b and error if the error is
       reduced
    if (abs(error_next - error_current) < tol) {
      break  # Stop if improvement is below the
         tolerance threshold
    } else {
      b_current <- b_next
      error_current <- error_next
    }
  }

  # Return the optimized b value
  return(b_current)

}
```

6.6 *b*-parameter

The loss aversion distribution is characterized by a unique parameter, b, which scales the distribution to appropriately fit the observed data accurately. k is a constant that delineates the domain within which the distribution is valid. Our analysis aims to explore the variations in b and x under the auspices of this newly characterized distribution. Notably, the probability density function associated with this distribution does not converge to $y = 0$ as x approaches infinity ($x_{0 \to \infty}$) thereby satisfying the conditions for being non-negative and monotonic. Furthermore, the integrals of the PDF over its defined domain are normalized to sum to unity, ensuring the distribution's applicability and validity within the specified parameters.

We introduce several candidates that could potentially explain the b parameter. First, it could be attributed to the time-varying preferences of consumers as shown below:

$$b = \beta_0 + \beta_1 e^{-\delta t}.$$

Initially, the preference for the goods is at its peak when t is close to 0. As t increases towards its maximum value, the effect of the β_1 parameter diminishes, leading to a decrease in consumer preference over time. In the context of loss aversion, the preference for the good is highest immediately after purchase, due to its novelty or recent acquisition, and gradually decreases as the food approaches its expiry or becomes less relevant. The second candidate for explaining the b parameter is the consumption rate modeled by logistic growth as shown below:

$$b(t) = \frac{K}{1 + e^{-r(t-t_0)}}.$$

When t is less than t_0, the change rate is minimal due to the large value in the denominator. As t approaches t_0, the change rate accelerates, marking a period of rapid growth. Beyond t_0, the rate of change decelerates and eventually stabilizes, indicating a shift to a more moderate or attenuated growth phase. In the context of loss aversion, the function describes how consumers' aversion to loss evolves over time after acquiring a good. Initially, loss aversion increases slowly as consumers start to feel ownership of the good. This growth in aversion accelerates when consumers recognize that the good is approaching its expiry. As the expiry becomes imminent, the increase in loss aversion slows down. Here, K represents the initial market price of the good at the time of acquisition, analogous to the constant k in the loss aversion function. The r parameter quantifies the rate at which loss aversion changes with each unit of time.

7 Assumptions

We posit that the variable K, representing the market price of a time-sensitive and value-depreciating good, remains constant over the period from ownership to expiry. This assumption may appear tenuous if the market value of such goods fluctuates significantly within this time frame. Nonetheless, the assumption gains validity when comparing goods of identical or nearly identical manufacture and expiry dates, leading us to adopt a strong stance that K remains invariant. Contrary to the assumption that the value of a time-sensitive and value-depreciating good could diminish to zero, we contend that such goods retain intrinsic value *ad infinitum*, attributed to the raw materials comprising them. Even post-expiry, the residual value of these materials is presumed to decline at an inconsequentially slow rate, underscoring the non-zero valuation of expired goods. This perspective aligns with the broader understanding that the fundamental worth of goods, especially in terms of their raw materials, diminishes marginally post-expiry, thereby challenging the notion of complete value depreciation.

8 Discussion

The loss aversion distribution characterizes consumer behavior towards perishable products, diverging from the traditional exponential family by exhibiting a non-memoryless property. This distribution details a nuanced pattern of loss aversion that begins with a muted response immediately after the manufacturing date. The aversion intensifies exponentially as the product approaches the midpoint between its manufacturing and expiry dates, before diminishing again towards the expiry date, culminating in a plateau of loss aversion behavior. For example, a chocolate bar manufactured one day prior and expiring a year later. The inclination to sell it at the original purchase price persists up to one month post-manufacture, reflecting a lack of urgency to adjust pricing. In contrast, the selling price undergoes a significant revision six months post-manufacture, mirroring heightened loss aversion as the expiry date draws closer, compared to when the product is eleven months from expiry. As ownership is at 11 months post-manufacture, loss aversion peaks, prompting a substantial price reduction in an effort to mitigate total loss. We substantiate the monotonic nature of this distribution through a derivative analysis of the probability density function (PDF), demonstrating that the derivative terms are negative, leading to monotonicity. Furthermore, we establish the boundedness of the function within the interval [0,1] and affirm its non-negativity through specific constraints. The non-memoryless characteristic of the loss aversion distribution is evidenced by the exponential decay's dependency on the variable x. Empirical evidence suggests that the b-parameter, upon sampling, aligns closely with the Gaussian distribution, as documented in Koh (2023). This insight, coupled with further research into psychometric assessments linked to the b-parameter, promises to enhance our understanding of loss aversion and consumer psychology significantly. Further research to expand the scope beyond perishable products can be promising, especially for collectible products.

Note

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